0755-1055 Friday 14 Dec. 2001

You may use a calculator. Write your name, alpha number and section on your blue book and the bubble sheet. Bubble in your alpha number in the left-most columns of the bubble sheet.

Part One. Multiple choice (50%). The first 20 problems are multiple choice. Fill in the best answer on the bubble sheet. There is no penalty for a wrong answer. YOU MUST ALSO PUT YOUR ANSWER AND SHOW ALL YOUR WORK IN THE BLUE BOOK(S).

---

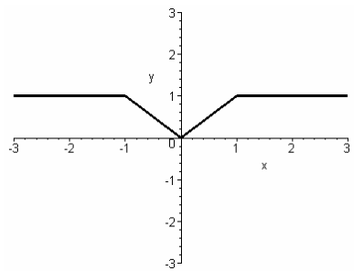
1. An equation of a circle with center at (2,-3) that passes through (2,1) is

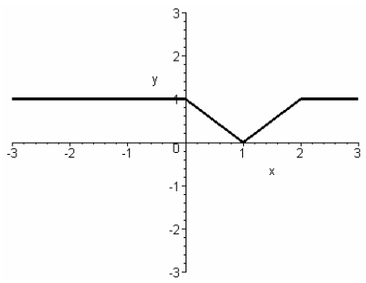
a) $x^2+y^2=16$ b) $(x-2)^2+(y+3)^2=4$ c) $(x-2)^2+(y+3)^2=16$

d) $(x+2)^2+(y-3)^2=4$ e) $(x+2)^2+(y-3)^2=16$

---

2. Given the graph of $f(x)$ on the left, the figure on the right is the graph of





a) $f(x-1)$ b) $f(x+1)$ c) $f(x)+1$ d) $f(x)-1$ e) none of these

---

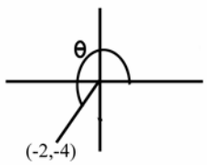
3. If $\log_a(x)=3$ and $\log_a(y)=4$ then $\log_a\left(\frac{x^2}{y}\right)=$

a) $\frac{9}{4}$ b) 2 c) 5 d) –2 e) 13

---

4. For the angle θ shown, $\tan(\theta)$ is

a) 1/2 b) 2 c) $-1/2$ d) –2 e) $-4/\sqrt{20}$



---

5. If $f(x)=\frac{4x-1}{2x+3}$ then $f^{-1}(x)$ (the inverse of f) is

a) $\frac{2x+3}{4x-1}$ b) $\frac{1+3x}{4-2x}$ c) $\frac{3x+1}{2x-4}$ d) $\frac{3+2x}{x-4}$ e) $\frac{2x+3}{1-4x}$

---

6. If $\lim_{x\to 2^-} f(x)=4$ $\lim_{x\to 2^+} f(x)=0$ and $f(2)=3$ then $\lim_{x\to 2} f(x)$ is

a) 3 b) 2 c) 4 d) 0 e) Does not exist

---

7. The function $h(t)=\begin{cases} 1/t & t<1 \\ t^2 & 1\le t\le 2 \\ t-1 & 2<t\le 3 \\ t^3 & 3<t \end{cases}$ is continuous at the point $t=$

a) 0 b) 1 c) 2 d) 3 e) None of these

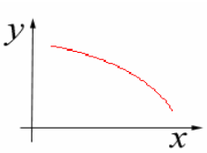
---

8. For the graph of $y=g(x)$ on the interval shown

a) $g'(x)>0, g''(x)=0$ b) $g'(x)>0, g''(x)>0$

c) $g'(x)>0, g''(x)<0$ d) $g'(x)<0, g''(x)<0$

e) $g'(x)<0, g''(x)>0$



9. The table below shows the percentage of a certain type of car still on the road after the car is discontinued. The average rate of change of the percentage between 12 and 24 months is

Months	6	12	18	24	30
Percentage	95	84	77	73	70

a) -11/12 %/Mo. b) -2/3 %/Mo c) 77/18 %/Mo d) 13/3 %/Mo e) –3/4%/Mo

---

10. The function $f(x) = \dfrac{3x^2+4}{x^2-4}$ has a horizontal asymptote of

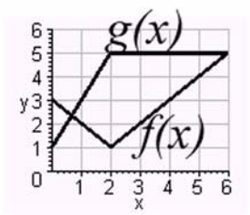
a) –2 b) 0 c) 1 d) 2 e) 3

---

11. For the functions whose graphs appear at the right,

$(f \circ g)'(1) =$

a) -2 b) 0 c) 1 d) 2 e) Does not exist



---

12. The equation of the line tangent to $y = \tan^{-1}(3x)$ at the point where $x = 0$ is given by

a) $y = \dfrac{3}{1+9x^2}x$ b) $y = \dfrac{1}{1+9x^2}x$ c) $y = -3x$

d) $y = 3x$ e) $y = \dfrac{540}{\pi}x$

---

13. Given that $f(1) = 3,\quad g(1) = 4,\quad f'(1) = 5$ and $g'(1) = -2$ then $\left(\dfrac{f}{g}\right)'(1) =$

a) $\dfrac{13}{8}$ b) $-\dfrac{13}{8}$ c) $\dfrac{7}{8}$ d) $-\dfrac{5}{2}$ e) $\dfrac{3}{4}$

---

14. The position of a ball thrown upward from a height of 64 feet is given by $p(t) = -16t^2 + 48t + 64$. The velocity when the ball hits the ground is

a) 0 ft/sec b) –20 ft/sec c) –40 ft/sec d) –60 ft/sec e) –80 ft/sec

---

15. Given the equation $x^2y + x^2 + y^2 = 3$, the slope of the line tangent to the curve at the point (1,1) is

a) $-4/3$ b) $-1/3$ c) $-1/2$ d) $-3/2$ e) $1/2$

---

16. For the function, $f(x) = 2x^3 - 15x^2 + 24x$ defined on the closed interval [0,6], The absolute minimum value of the function occurs at the point where x=

a) 0 b) 1 c) 2 d) 4 e) 6

17. The absolute maximum value of the same function as in problem 16 occurs at the point where x=

a) 0 b) 1 c) 2 d) 4 e) 6

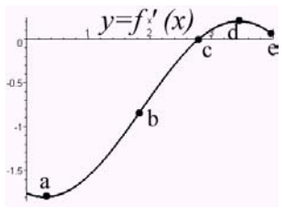
---

18. The function $f(x) = xe^{-x/2}$ has a point of inflection at the point where x=

a) 0 b) 1 c) 2 d) 4 e) there is no point of inflection

19. The graph pictured to the right is the graph of $f'(x)$ not $f(x)$. The function, $f(x)$, has its least value at the point on the graph labeled

a b c d e



20. For the same graph as problem 19, $f''(x)$ has its greatest value at the point on the graph labeled

a b c d e

Part Two. Longer Answers(50%). Do the remaining 10 problems. They are not multiple choice. Show all work and put your answers in your blue book(s).

---

21. For a certain type of radioactive compound, it is found that 10% decays in 20 years.

a) How much of the substance is left after 40 years, if the initial amount is 10 grams?

b) Find a formula for the amount of the compound present after *t* years.

c) Determine the half-life of the compound. In other words, the length of time until 50% of the substance has decayed.

---

22. a) Sketch the graph of the curve defined by $x = 3\cos(t), \quad y = 4\sin(t), \quad 0 \le t \le 2\pi$

b) Find the equation of the line tangent to this curve at the point where $t = \frac{\pi}{4}$.

---

23. a) What is the limit definition for $f'(x)$?

b) Use the limit definition to show that $f'(x) = 4x - 4$ for $f(x) = 2x^2 - 4x + 3$

---

24. a) What is the domain of the function $g(x) = \frac{\sqrt{x+1}}{x}$?

b) If $f(x) = x^2 + 1$, find a formula for $g(f(x))$.

---

25. a) The Centers for Disease Control have determined that 10000 people have been infected by a certain virus and the number is increasing by 500 per day. What is the linear approximation for the number of people infected?

b) Estimate the number of people we expect to be infected 3 days later.

c) If we know that the rate of at which new people are being infected is decreasing, is the approximation in part b) an over approximation or an under approximation?

---

26. The graph of the function, $f(x)$, appears to the right.

a) On what intervals is $f'(x) > 0$ and on what intervals is $f'(x) < 0$?

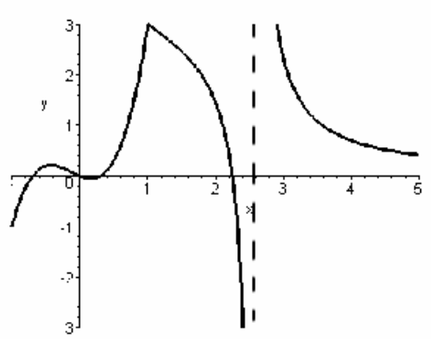
b) Where are the local maximas and minimas?

c) On what intervals are $f''(x) > 0$ and on what intervals are $f''(x) < 0$?

d) Where are the points of inflection?

e) Where are the vertical asymptotes?

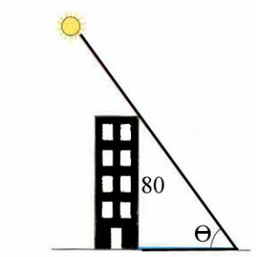
f) Sketch the graph of $f'(x)$.



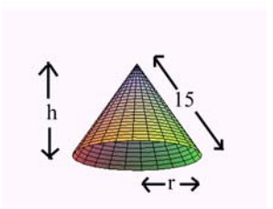
---

27. Use L'Hopital's rule to find $\lim_{x \to 0} \frac{1 - \cos(x)}{x\sin(x)}$. You must show your work for each step of the problem.

28. On a day that the sun will pass directly overhead, the sun is at an angle of elevation, $\theta = \pi/4$ at 9am. The angle of elevation is increasing at the rate of $\frac{2\pi}{24 \bullet 60}\frac{\text{rad}}{\text{min}}$. How is the length of the shadow cast by an 80-foot tall building changing? See the diagram to the right.



29. What are the dimensions of the cone with slant height 15 meters that has the largest volume? You may use the formula $V = \frac{\pi r^2 h}{3}$.



30. Sketch the graph of a single function, $f(x)$ which satisfies the conditions given below.

$\lim_{x\to 2} f(x) = \infty$, $\lim_{x\to\pm\infty} f(x) = 1$

$f'(x) < 0$ on $(-\infty,0)$ and $(2,\infty)$ $f'(x) > 0$ on $(0,2)$ $f'(0) = 0$

$f''(x) < 0$ on $(-\infty,-2)$ $f''(x) > 0$ on $(-2,2)$ and $(2,\infty)$ $f''(-2) = 0$

$f(0) = -3$ $f(-2) = 0$